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| Mathematics Design Collaborative |
| State of Georgia Department of Education |



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| Understanding probability: types of events |
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| *Compound Confusion* |
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| INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON |

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| **MATHEMATICAL GOALS:** |
| This lesson unit is intended to help you assess how well students are able to: |
| * Utilize what they already know about compound probability in the context of real world situations. * Reason qualitatively, choose and apply the correct type of compound probability to the given situation. * Understand the meaning of independent, dependent, mutually exclusive, conditional, and overlapping probability. * Solve scenarios using independent, dependent, mutually exclusive, conditional, and overlapping probability. |

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| **GEORGIA STANDARDS OF EXCELLENCE:** |
| This lesson involves mathematical content in the standards from across the grades, with emphasis on:  **MGSE9-12.S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*  SMP1. Make sense of problems and persevere in solving them.  SMP3. Model with mathematics. |
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| **INTRODUCTION:** |
| This lesson is structured in the following way: |
| Before the Lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions. |
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| During the Lesson, students will work in pairs or threes to match fifteen real world scenarios with the appropriate type of compound probability. The students must give reasons for why they made each selection. As an extension, students will solve the probability scenarios using the appropriate type of compound probability. |
| After the Whole-Group Class Discussion, students will re-visit the *Compound Probability* task, and demonstrate what they have learned from the lesson. |

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| **MATERIALS REQUIRED:** |
| Each individual student will need: |
| * A copy of the worksheet *Compound Probability* for both the pre-assessment and post-assessment (2 per student).   Each group will need: |
| * Set of card set A * Set of card set B * Poster or printer paper * Markers, Glue sticks or tape, Pencils, Notebook paper |

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| **TEACHER PREP REQUIRED:** |
| Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made: |
| * 2 copies per student of the *Compound Probability* task * Card set A and card set B for each group, already cut out |

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| **TIME NEEDED:** | | | | |
| For Pre-Assessment: 15 minutes |  | For Lesson: 45 minutes | For Post: 15 minutes |
| Special Note(s) about timing: All timings are approximate. If there is not enough time to complete pre or post assessment in class, the teacher may assign it for homework. | | | | |

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| **FRAMING FOR THE TEACHER:** |
| This lesson is intended to assess students’ understanding of probability and the types of events. This knowledge will later be used to solve real world probability problems. This lesson should be given after the students have been introduced to the formulas for solving each type of probability. The students should write notes from the class discussion on common characteristics of each type of probability so that they have resources to reference when working in pairs. This FAL should eradicate any misconceptions students may have in determining what type of event is occurring in each problem, and relate them to real world situations. |
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| **FRAMING FOR THE KIDS:** |
| Say to the students: |
| *This activity will take about 1 day for us to complete.* |
| *The reason we are doing this is to be sure that you understand types of events of probability before we move on to a new idea.* |
| *You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!* |

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| PRE-ASSESSMENT BEFORE THE LESSON |
| **ASSESSMENT TASK:** |
| Name of Assessment Task: *Compound Probability* |
| Time This Should Take: 15 minutes |

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.



Give each student a copy of *Compound Probability*

**Compound Probability**

Briefly introduce the task and help the class to understand the problem and its context.

*Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

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| COLLABORATION TIME/READING STUDENTS RESPONSES |
| **You Will Not “Grade” These!** |
| Collect students’ responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!) |
| You will begin to construct Socratic-style questions to try and elicit understanding from students. We suggest you write a list of your own questions however some guiding questions and prompts are also listed below as a jumping-off point. |

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| **GUIDING QUESTIONS** | |
| ***COMMON ISSUES*** | ***SUGGESTED QUESTIONS AND PROMPTS*** |
| Student Was Unable to Begin Assessment  *Example: “I don’t know recognize any of these words.” “I don’t know anything about compound probability.”* | * What does the term probability mean? * How do you calculate basic probability? * What does probability have to do with chance? |
| Student couldn’t write the correct meaning to one or more of the types of compound probability.  *Example: “Independent probability is where events rely on each other.”* | * What does the word “independent” mean in independent probability? * What does the word “dependent” mean in dependent probability? * What does it mean if something is “conditional”? * If something is exclusive, does it depend on something else? * If something overlaps, does it include something else? |
| Student couldn’t match one or more of the situations with the correct probability.  *Example: “Scenario C is matched with Mutually Exclusive Probability.”* | * What key words in each scenario lead you to tell the type of probability it is? * Does the scenario rely on something else? |
| Student couldn’t explain why they chose the situation to match the correct type of probability.  *Example: “I don’t know why I chose the answer I chose.”* | * What key words led you to your decision? * What is the meaning of each type of probability? * How did you know where to begin? |
| Student was unable to calculate the correct probability. *Example: “I don’t know how to calculate probability.”* | * What type of probability are you working with? * What are the basic steps to calculating probability? * What are the different steps between each type of probability? |

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| LESSON DAY | |
| **SUGGESTED LESSON OUTLINE:** | |
| **Part 1: Whole-Class Introduction:** | **Time to Allot: ( 10 minutes)** |
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| Write on the board the words *independent probability, dependent probability, conditional probability, overlapping probability,* and *mutually exclusive probability.* Next, have students tell you characteristics of each type of probability and write them under each type. Then, discuss as a class how to tell the difference between each type of probability and the key words to look for.  Suggested Questions:  *What are the characteristics of each type of probability written on the board?*  *What features distinguish one type of probability from another?*  *When reading a word problem, what key words aid you in determining which type of probability to use to solve for the answer?*  *What formulas do you use to solve each type of probability?* | |
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| **Part 2: Collaborative Activity:** | **Time to Allot: ( 40 minutes)** |
| Put students into their pairs according to your analysis of student errors. | |
| Do/Say the Following: | |
| |  |  |  | | --- | --- | --- | | Group students in pairs by common errors found in the Compound Confusion task.  Students are given cards set A and B, already cut apart.  Each group should match the cards in set B with one card from set A.  \*\*The last page of the packet summarizes the instructions for this activity.  During the Collaborative Activity, the Teacher has 3 tasks:   * Circulate to students whose errors you noted from the pre-assessment and support their reasoning with your guiding questions. * Circulate to other students also to support their reason in the same way. * Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!   If you find students have difficulty articulating their decisions, use the sheet Suggested Questions and Prompts to support your own questioning of students.  *Sample ways to jump-start students’ work in the group collaboration:*   |  | | --- | | *Can you find a situation where you definitely know which type of probability it belongs to?* | | *What are the main differences between each probability?* |   **Collaborative Activity Part 2: Sharing or Presenting Time: (20 minutes)**  Each pair will choose one student to remain at their station and one student to rotate around the room.  Students will compare their answers with the other stations.  The students will provide an explanation as to why their pairs chose the answers they chose. | | |
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| **Part 3: Plenary (Summary) Discussion:** | **Time to Allot: ( 15 minutes)** |
| Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below: | |
| NOTE: *“Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!* | |
| Bring the class together and use the following to guide discussion:  Students will present their charts.  *What key words in the situation told you what type of probability it was?*  *What is the difference between conditional probability and mutually exclusive probability?*  *What is the difference between independent and dependent probability?*  *What do you notice about overlapping probability and the other types of probability mentioned?*  *What steps would you take to solve each of the probabilities mentioned?*  *What do the numbers in the situation represent?* | |

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| **Part 4: Improving Solutions to the Assessment Task** | **Time to Allot: ( 20 minutes)** |
| The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.  Practice will make perfect, and teachers should do what makes them most comfortable with their students. | |
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| PRE-ASSESSMENT (Answer Key) |
| **ASSESSMENT TASK:** |
| Name of Assessment Task:Compound Confusion |
| Part 1: (a) type of probability in which the fact that event A occurring doesn’t affect the probability of event B occurring (b) type of probability in which the occurrence of event A affects the occurrence of event B (c) probability of an event occurring given that another event has occurred (d) when two events can’t occur at the same time (e) when two events can occur together  Part 2: (a) Conditional, P(A/B) (b) Overlapping, P(A or B) (c) Mutually Exclusive, P(A or B) (d) Dependent Probability, P(A and B) (e)Independent Probability, P(A and B)  Part 3: (a) Answers will vary. (b) Question A: 63/83; Question B: 4/7; Question C: 2/9; Question D: 1/44; Question E: 64/225 |

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| Collaborative Activity (Answer Key) |
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| P(A and B) Independent: Cards 1 & 5  P(A and B) Dependent: Cards 3 & 7  P(A/B) Conditional: Cards 4, 6, & 8  P(A or B) Mutually Exclusive: Cards 2 & 1    P(A or B) Overlapping: Cards 9, 10, & 12  10 |

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| PRE-ASSESSMENT |
| Name of Assessment Task: Compound Probability |
| 1. **State a definition for each of the following types of probability**:   A. Independent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  B. Dependent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  C. Conditional \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  D. Mutually Exclusive \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  E. Overlapping \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. **Match each of the following situations with one of following: P(A and B) Independent**, **P(A and B) Dependent, P(A/B) Conditional, P(A or B) Mutually Exclusive, P(A or B) Overlapping.** 2. In Georgia, 83% of 16 year-olds have a cell phone and 63% have a cell phone and a car. What is the probability that a teenager has a car given that he or she also has a cell phone? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3. Maggie studies with a group for an upcoming math competition on Mondays, Tuesdays, and Thursdays. She also volunteers at a hospital on Mondays, Wednesdays, and Thursdays. Maggie’s science class is taking a field trip that could be scheduled for any day of the week (Monday through Friday). Find the probability that the field trip will be scheduled for a day that Maggie is studying for her math competition or volunteering at the hospital. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 4. Johnny is playing a game and rolls a pair of dice. What is the probability that the sum of the dice rolled is either a 9 or a 5? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 5. Carrie writes each of the letters of the word MATHEMATICAL on individual index cards and places them into a bag. She randomly draws one letter from the bag, doesn’t replace it, and then randomly draws a second letter. What is the probability that the first letter is an “A” and the second letter is a “H” ? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 6. A bag contains 8 orange balls and 7 purple balls. Josh randomly draws one ball replaces it, and randomly draws a second ball. What is the probability of the first ball being orange and the second ball being orange? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 7. **Pick three of the situations in problem 2 and answer the following:** 8. Explain the reasoning behind the solutions you chose.      1. Solve the three scenarios you chose to find the actual probabilities. |

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| COLLABORATIVE ACTIVITY |
| Name of Assessment Task: |
| ***Card Set A: Probabilities***   |  |  | | --- | --- | | **P(A and B) Independent** | **P(A and B) Dependent** | | **P(A/B) Conditional** | **P(A or B) Mutually Exclusive** | | **P(A or B) Overlapping** |   ***Card Set B: Situations***   |  |  | | --- | --- | | **Card 1: A jar of coins contains 3 quarters, 8 dimes, 6 nickels, and 5 pennies. A coin is chosen at random from a jar. After replacing it, a second coin is chosen from the same jar. What is the probability of choosing a dimes the first time and a penny the second time?** | **Card 2: A box contains 20 red, 10 blue, and 30 yellow beads. What is the probability of a bead drawn at random being red or blue?** | | **Card 3: A card is chosen at random from a standard deck of 52 cards. Without replacing the first card, a second card is chosen at random. What is the probability that both cards will be a spade?** | **Card 4: At Ware County High School, the probability that a student takes Technology and Spanish is 0.076. The probability that a student takes Technology is 0.70. What is the probability that a student takes Spanish given that they are taking Technology?** | | **Card 5: Sally was given a standard deck of 52 cards. She randomly chose 3 cards from the deck replacing the card before each time. What is the probability of choosing an ace, a queen, and a six?** | **Card 6: The probability that it is Friday and a student is absent is 0.13. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent since it is Friday?** | | **Card 7: At the tire store, 10 out of every 100 tires are defected. If your parents randomly choose and purchase 4 new tires for a family vehicle from a set of 100 newly shipped tires, what is the probability that all four tires will be defective?** | **Card 8: A jar contains red and blue marbles. Two marbles were chosen without replacement. The probability of selecting a red marble and then a blue marble is 0.26, and the probability of selecting a blue marble first is 0.43. What is the probability of selecting a red marble on the second draw, given that the first marble drawn was a blue marble?** |  |  |  | | --- | --- | | **Card 9: The letters of the word “THOUGHT” and “TIME” are written on individual cards and place into a bag. A card is picked at random. What is the probability of picking an “E” or a “T”?** | **Card 10: A certain manufacturer of cake, muffin, and bread mixes has 100 buyers, 50 of whom by cake mix, 40 buy muffin mix, and 20 buy both cake and muffin mix. If a buyer is to be selected at random from 100 buyers, what is the probability that the buyer will be one who purchases neither cake or muffin mix?** | | **Card 11:** **There are 3 literature books, 4 algebra books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature books or an algebra book?** | **Card 12: In the Math Club, 7 of the 20 girls are seniors, and 4 of the 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest?** | |

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| **Collaborative Activity Instructions:** |
| 1. You have been grouped in pairs. 2. You are given card sets A (each type of compound probability) and B (each probability situation), already cut apart. 3. Read the situations very critically and very carefully. You and your partner should match each situation from card set B to each type of compound probability from card set A. Discuss to ensure that you both agree. Each type may have more than one situation. 4. Once you are sure that you have completed the matching correctly, begin solving each of the situations to find the correct probability. 5. Grab a sheet of chart paper and glue each situation under the correct probability heading.   6) Be prepared to justify your answers and to discuss |